An Algorithm for graceful labelings of certain unicyclic graphs and some new graceful $C_{2n+1}$-unicyclic graphs

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A graceful labeling of a simple graph $G$ is an injection $f$ from the vertices of $G$ to the set $\{0,1,2,\cdots,|E(G)|\}$, such that when each edge $xy$ is assigned the label $|f(x) - f(y)|$, the resulting edge set is $\{1,2,\cdots,|E(G)|\}$, with no label repeated. We pay attention to Truszczynski’s conjecture, that all unicyclic graphs except cycles $C_n$ with $n \equiv 1(mod\ 4)$ or $n \equiv 2(mod\ 4)$, are graceful. Jay Bagga et al. introduced an algorithm to enumerate graceful labelings of cycles and ”sun graphs”. We generalize their algorithm to enumerate all graceful labelings of a class of irregular unicyclic graphs and provide some experimental results. We also present some new graceful unicyclic graphs, $C_{2n+1} \oplus (R(u); R'(v); P_{k_1}; P_{k_2}; \cdots; P_{k_{2n-1}})$, where $k_i \in \{1,2\}$, $P_n$ is a path of length $n$, each path $P_i$ sharing one peripheral vertex with $C_{2n+1}$, $R$ and $R'$ are caterpillars sharing respectively vertices $u$ and $v$, peripheral vertices or vertices neighbors of peripheral vertices, with $C_{2n+1}$, $u$ and $v$ being neighbors in the cycle. We show at the end how our work contributes to the proof of the conjecture of Truszczynski.

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