4-Chromatic Subgraphs of $\mathbb{Q}^4$

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A bit of notation – for $X \subset \mathbb{R}^n$ and $d > 0$, let $G(X, d)$ denote the distance graph with vertex set $X$ where any two vertices are adjacent if and only if they are a Euclidean distance $d$ apart. Chromatic numbers and related properties of $G(\mathbb{Q}^n, d)$ have been studied with varying degrees of success since the 1970s. In a recent paper, Peter D. Johnson showed that four colors are sufficient to properly color $G(\mathbb{Q}^4, d)$ for any $d > 0$. In this talk we will use a few results from classical number theory to show that four colors are also necessary to properly color any non-trivial graph $G(\mathbb{Q}^4, d)$. We then close with a few open problems related to the subject.

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