The number of mates of latin squares of sizes 7 and 8

Megan Bryant*, James Figler, Roger Garcia, Carl Mummert, Yudishthisir Singh, Marshall University

We study the number of mates that a latin square may possess as a function of the size of the square. We performed an exhaustive computer search of all squares of sizes 7 and 8, giving the exact value for the maximum number of mates for squares of these sizes. The squares of size 8 with the maximum number of mates are exactly the Cayley tables of $\mathbb{Z}_2^3 = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, and each such square has $70,272 \cdot 8!$ mates. We obtain a combinatorial proof that, for every $k \geq 2$, the square obtained from a Cayley table of $\mathbb{Z}_2^k$ has a mate. This research was partially supported by NSF grants OCI-1005117 and EPS-0918949.

Keywords: latin square, orthogonal, mates