Extended Skolem-type Difference Sets

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A k-extended Skolem-type 5-tuple difference set of order t is a set of t 5-tuples
\(\{ (d_{i,1}, d_{i,2}, d_{i,3}, d_{i,4}, d_{i,5}) \mid i = 1,2, \ldots, t \}\) such that
\(d_{i,1} + d_{i,2} + d_{i,3} + d_{i,4} + d_{i,5} = 0\) for \(1 \leq i \leq t\)
and \(\{|d_{i,j}| \mid 1 \leq i \leq t, 1 \leq j \leq 5\} = \{1,2, \ldots, 5t+1\} \setminus \{k\}\). In this talk, we will give necessary and sufficient conditions on \(t\) and \(k\) for the existence of a \(k\)-extended Skolem-type 5-tuple difference set of order \(t\). We also consider hooked \(k\)-extended Skolem-type 5-tuple difference sets of order \(t\) and provide necessary and sufficient conditions for their existence. We will then show how these \(k\)-extended Skolem-type difference sets can be used to find decompositions of circulant and complete graphs of order \(n\) into 5-cycles, \(d\)-cycles, where \(d\) is a divisor of \(n\), Hamilton cycles, and possibly a 1-factor.

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