On Morphisms and Nested Recurrence Relations

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We explore a family of recurrence relations with arbitrary levels of nesting. These recurrences have a combinatorial interpretation in terms of fixed points of morphisms over a countably infinite alphabet. We analyze the asymptotic behaviour of such sequences by applying known results on the frequency of letters in morphic words. Recurrences in this family are related to a number of well-known sequences, including Hofstadter’s G sequence, the Conolly and Tanny sequences, and the Thue-Morse sequence.

Keywords: morphism, nested recurrence relation, meta-Fibonacci sequence.