Some Results on the Radon Number of Graphs

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A graph convexity is a pair \((V(G), \mathcal{C})\), where \(V(G)\) is the vertex set of some graph \(G\) and \(\mathcal{C}\) is a collection of subsets of \(V(G)\), being closed under intersections and containing \(V(G)\) itself and the empty set. The sets of \(\mathcal{C}\) are called convex. For a subset \(S \subseteq V(G)\), the hull of \(S\), denoted \(H(S)\) is the (unique) smallest convex set containing \(S\). Inspired by the celebrated Radon’s theorem (1921), the Radon number of a graph has been defined as the smallest integer \(k\), such that every set \(R \subseteq V(G)\) of size at least \(k\), admits a bipartition \(R = R_1 \cap R_2\) satisfying \(H(R_1) \cap H(R_2) \neq \emptyset\). We describe results on the Radon number of graphs, considering the \(P_3\) convexity, that is, the one whose convex sets are closed under common neighbors. These results include complexity, the computation of the exact Radon number for some graph classes and general bounds.

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