Grundy Numbers of Strong Products of Graphs

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A proper $k$-coloring $c$ of a graph $G$ is called Grundy $k$-coloring if for each vertex $u$ and all colors $i$ with $1 \leq i < c(u)$ there exists an adjacent vertex $v$ with $c(v) = i$. The maximum number $k$ of colors for which a Grundy $k$-coloring using all $k$ colors exists is the Grundy number $\Gamma(G)$ of $G$.

The strong product of graphs $G = (V_G, E_G)$ and $H = (V_H, E_H)$ has as vertex set the Cartesian product $V_G \times V_H$ of the vertex sets of $G$ and $H$. Two vertices $(u, v)$ and $(\bar{u}, \bar{v})$ of $V_G \times V_H$ are adjacent if and only if $u = \bar{u}$ and $v\bar{v} \in E_H$ or $v = \bar{v}$ and $u\bar{u} \in E_G$ or $u\bar{u} \in E_G$ and $v\bar{v} \in E_H$.

We completely determine the Grundy numbers of strong products of paths and cycles.

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