Nordhaus-Gaddum Theorems for $k$- Decompositions

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Consider the following problem. An international round-robin sports tournament is held between teams. The games are split between locations in different countries, which can host multiple games simultaneously. The teams can travel to different locations to play, but it is impractical for the fans to visit more than one location. In this situation, it is reasonable to want teams that play at a given location to play as many games there as possible so that local fans can see them as much as possible. More precisely, we can compute the minimum number of games played by the teams at that location. We then wish to maximize the sum of these minimum numbers over all the locations in the tournament.

Thus we wish to find the maximum sum of the minimum degrees over all possible decompositions of a graph into $k$ factors. This is a generalization of a Nordhaus-Gaddum Theorem, which finds sharp upper and lower bounds for the sum of a parameter on a graph and its complement. Furedi, Kostochka, Stiebitz, Skrekovski, and West [2005] determined some bounds and exact values of this quantity for $k = 2, 3, 4$. We extend these results by determining all possible decompositions achieving these bounds and considering larger values of $k$.

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