Distance Distribution of Trees

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Let $T$ be a tree of order $n$ and $k$ a non-negative integer. We determine best possible upper and lower bounds on $D_k(T)$, the number of pairs of vertices at distance exactly $k$ in $T$. We show that for odd $k$,

$$D_k(T) \leq \left\lfloor \frac{n - k + 1}{2} \right\rfloor \left\lceil \frac{n - k + 1}{2} \right\rceil,$$

and for even $k$

$$D_k(T) \leq \begin{cases} 
\binom{n-1}{2} & \text{if } k = 2, \\
\frac{1}{2}n^2 - \sqrt{k - 2n^{3/2}} + O(n) & \text{if } k \geq 4,
\end{cases}$$

We also give bounds on $D_k(T)$ in terms of order and radius or diameter.

Keywords: distance, diameter, radius, tree