A Proof from the Book: A Lower Bound for the Polychromatic Number of the Plane
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We all have a tendency to stuff too much information and not enough proofs in our talks. Let me try to do the opposite: just show one remarkably beautiful proof, found by the Moscow high school student Alexei Merkov.

*Polychromatic number of the plane* is the minimum number of colors needed for coloring the plane in such a way that no color realizes all distances. The lower bound of 4 was first obtained in 1970 by another Moscow high school student Dmitry Raiskii. Merkov’s 1997 proof comes straight from The Book, as Paul Erdős would have undoubtedly said about it. I have gently improved the narrative of the proof.

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