The Competition-common Enemy Graphs of Digraphs Satisfying the Conditions $C(p)$ and $C'(p)$

Yoshio Sano, Kyoto University

S. -R. Kim and F. S. Roberts (2002) introduced the following conditions $C(p)$ and $C'(p)$ for digraphs as generalizations of the condition for digraphs to be semiorders. The condition $C(p)$ (resp. $C'(p)$) is: For any set $S$ of $p$ vertices in $D$, there exists $x \in S$ such that $N_D^+(x) \subseteq N_D^+(y)$ (resp. $N_D^-(x) \subseteq N_D^-(y)$) for all $y \in S$, where $N_D^+(x)$ (resp. $N_D^-(x)$) is the set of out-neighbors (resp. in-neighbors) of $x$ in $D$. The competition graph of a digraph $D$ is the (simple undirected) graph which has the same vertex set as $D$ and has an edge between two distinct vertices $x$ and $y$ if $N_D^+(x) \cap N_D^+(y) \neq \emptyset$. Kim and Roberts characterized the competition graphs of digraphs which satisfy the condition $C(p)$.

The competition-common enemy graph of a digraph $D$ is the graph which has the same vertex set as $D$ and has an edge between two distinct vertices $x$ and $y$ if it holds that both $N_D^+(x) \cap N_D^+(y) \neq \emptyset$ and $N_D^-(x) \cap N_D^-(y) \neq \emptyset$. In this talk, we give a characterization for the competition-common enemy graphs of digraphs satisfying the conditions $C(p)$ and $C'(p)$.

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