On $r$-locating-dominating sets in cycles and paths

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The concept of locating-dominating sets in graphs was first introduced by P. J. Slater in the 1980’s. Bertrand, Charon, Hudry and Lobstein (2004) studied $r$-locating-dominating sets in cycles $C_n$ and paths $P_n$. They conjectured that if $r \geq 2$ is a fixed integer, then the smallest cardinality of an $r$-locating-dominating set in $P_n$, denoted by $M_{rLD}(P_n)$, satisfies $M_{rLD}(P_n) = \lceil (n+1)/3 \rceil$ for infinitely many values of $n$. We prove that this conjecture holds. In fact, we show a stronger result saying that for any $r \geq 5$ we have $M_{rLD}(P_n) = \lceil (n+1)/3 \rceil$ for all $n \geq n_r$ when $n_r$ is large enough ($n_r = O(r^3)$). Moreover, we determine the exact values of $M_{3LD}(P_n)$ and $M_{4LD}(P_n)$ for all positive integers $n$. Previously, the exact values of $M_{2LD}(P_n)$ have been solved by Honkala (2009).

Chen, Lu and Miao (2009) determined the smallest cardinalities $M_{2LD}(C_n)$ of 2-locating-dominating sets in $C_n$ for all positive integers $n$. We concentrate here on the smallest cardinalities $M_{rLD}(C_n)$ of $r$-locating-dominating sets in $C_n$ with the radius $r \geq 3$. First of all, we prove that for any $r \geq 5$ and $n \geq n_r$ when $n_r$ is large enough we have $M_{rLD}(C_n) = \lceil n/3 \rceil$ if $n \not\equiv 3 \pmod{6}$ and $n/3 \leq M_{rLD}(C_n) \leq n/3 + 1$ if $n \equiv 3 \pmod{6}$. The exact values of $M_{3LD}(C_n)$ and $M_{4LD}(C_n)$ are also determined for all positive integers $n$. In particular, we show that if $n \equiv 3 \pmod{6}$, then $M_{3LD}(C_n) = n/3 + 1$ and $M_{4LD}(C_n) = n/3 + 1$ for all $n \geq 15$.

Keywords: Locating-dominating set; locating-dominating code; domination; cycle; path