On Jacobsthal Binary Sequences

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Let $\Sigma = \{0, 1\}$ be the binary alphabet, and $A = \{0, 01, 11\}$ the set of three strings 0, 01, 11 over $\Sigma$. Let $A^*$ denote the Kleene closure of $A$, and $\mathbb{Z}^+$ the set of positive integers. A sequence in $A^*$ is called a Jacobsthal binary sequence. The number of Jacobsthal binary sequences of length $n \in \mathbb{Z}^+$ is the $n^{th}$ Jacobsthal number. Let $k \in \mathbb{Z}^+$, $1 \leq k \leq n$. The number of Jacobsthal binary sequences with 1 at the $k^{th}$ position from the left is denoted by $a_{n,k}$. A formula for this number has been derived recently. In this paper we consider the general case of $a(n; k_1, k_2, \ldots, k_m)$, the number of Jacobsthal binary sequences with 1 at each of the $k_i^{th}$ ($1 \leq i \leq m$) positions from the left, where $m, k_i \in \mathbb{Z}$; $1 \leq m < n$; $1 \leq k_1 < k_2 < \ldots < k_m \leq n$. We present a formula for $a(n; k_1, k_2, \ldots, k_m)$, and study some other special types of Jacobsthal binary sequences. Some identities involving these numbers are also given.

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