Recognition Algorithms and Structural Characterizations for Bipartite Tolerance and Bipartite Probe Interval Graphs

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A graph $G$ is a tolerance graph if and only if each vertex $v \in V(G)$ can be associated with an interval $I_v$ of the real numbers and a positive real number $t_v$ with $uv \in E(G)$ if and only if $|I_v \cap I_u| \geq \min\{t_v, t_u\}$. Graph $G$ is a probe interval graph if there is a partition of $V(G)$ into sets $P$ and $N$ with each vertex associated to an interval of the real number line such that $uv \in E$ if and only if $I_u \cap I_v \neq \emptyset$ and $\{u, v\} \cap P \neq \emptyset$. We give a recognition algorithm for bipartite tolerance graphs that yields a structural characterization in terms of 2-connected blocks. With a few modifications, the same recognition algorithm works for bipartite probe interval graphs and yields a structural characterization for them in terms of 2-edge-connected blocks. The recognition algorithm is $O(|V| + |E|)$ for both classes of graphs.