The 3-colored Ramsey Number of Even Cycles

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Denote by $R(L, L, L)$ the minimum integer $N$ such that any 3-coloring of the edges of the complete graph with $N$ vertices $K_N$ contains a monochromatic copy of a graph $L$. Bondy and Erdős conjectured that for an odd $n$-cycle $C_n$, $R(C_n, C_n, C_n) = 4n - 3$ for $n > 3$. This is sharp if true. Luczak proved that if $n$ is odd, then $R(C_n, C_n, C_n) = 4n + o(n)$, as $n \to \infty$. Kohayakawa, Simonovits and Skokan proved that the exact Bondy-Erdős conjecture holds for sufficiently large values of $n$. Figaj and Luczak determined an assintotic result for the ‘complementary’ case where the cycles are even: they showed that for $n$ even $R(C_n, C_n, C_n) = 2n + o(n)$ (Actually their result is much stronger than that because the cycles may be of slightly different sizes). Now we prove that there is $n_0$ such that for $n \geq n_0$ even $R(C_n, C_n, C_n) = 2n$.

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