Maximum packing for perfect four-triple configurations

Selda Küçükcifçi and Güven Yücetürk*
Auburn University

The graph consisting of the four 3-cycles (triples) \((x_1, x_2, x_8), (x_2, x_3, x_4), (x_4, x_5, x_6),\) and \((x_6, x_7, x_8),\) where \(x_i\)’s are distinct, is called a 4-cycle-triple block and the 4-cycle \((x_2, x_4, x_6, x_8)\) of the 4-cycle-triple block is called the interior (inside) 4-cycle. The graph consisting of the four 3-cycles \((x_1, x_2, x_6), (x_2, x_3, x_4), (x_4, x_5, x_6),\) and \((x_6, x_7, x_8),\) where \(x_i\)’s are distinct, is called a kite-triple block and the kite \((x_2, x_4, x_6)\)-\(x_8\) (formed by a 3-cycle with a pendant edge) is called the interior kite. A decomposition of \(3kK_n\) into 4-cycle-triple blocks (or into kite triple blocks) is said to be perfect if the interior 4-cycles (or kites) form a \(k\)-fold 4-cycle system (or kite system). A packing of \(3kK_n\) with 4-cycle-triples (or kite-triples) is a triple \((X, B, L)\), where \(X\) is the vertex set of \(K_n\), \(B\) is a collection of 4-cycle-triples (or kite-triples), and \(L\) is a collection of 3-cycles, such that \(B \cup L\) partitions the edge set of \(3kK_n\). If \(|L|\) is as small as possible, or equivalently \(|B|\) is as large as possible, then the packing \((X, B, L)\) is called maximum. If the maximum packing \((X, B, L)\) with 4-cycle-triples (or kite-triples) has the additional property that the interior 4-cycles (or kites) plus a specified subgraph of the leave \(L\) form a maximum packing of \(kK_n\) with 4-cycles (or kites), it is said to be perfect.

This paper gives a complete solution to the problem of constructing perfect maximum packings of \(3kK_n\) with 4-cycle-triples and kite-triples, whenever \(n\) is the order of a \(3k\)-fold triple system.

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