$[r, s, t]$-Chromatic Numbers of Graphs

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Given non-negative integers $r$, $s$, and $t$, an $[r, s, t]$-coloring of a graph $G$ with vertex set $V(G)$ and edge set $E(G)$ is a mapping $c$ from $V(G) \cup E(G)$ to the color set $\{0, 1, \ldots, k-1\}$ such that $|c(v_i) - c(v_j)| \geq r$ for every two adjacent vertices $v_i, v_j$, $|c(e_i) - c(e_j)| \geq s$ for every two adjacent edges $e_i, e_j$, and $|c(v_i) - c(e_j)| \geq t$ for all pairs of incident vertices and edges, respectively. The $[r, s, t]$-chromatic number $\chi_{r,s,t}(G)$ of $G$ is defined to be the minimum $k$ such that $G$ admits an $[r, s, t]$-coloring.

This is an obvious generalization of all classical graph colorings since $c$ is a vertex coloring if $r = 1$, $s = t = 0$, an edge coloring if $s = 1$, $r = t = 0$, and a total coloring if $r = s = t = 1$, respectively.

We present general bounds for $\chi_{r,s,t}(G)$ as well as exact values for certain parameters. Moreover, we completely determine the $[r, s, t]$-chromatic numbers for stars.

Keywords: Chromatic number, chromatic index, total chromatic number
MSC: 05C15