Degree bounded factorizations of pseudographs

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For \( d \geq 1, s \geq 0 \) a \((d, d+s)\)-graph is a graph whose degrees all lie in the interval \( \{d, d+1, \ldots, d+s\} \). For \( r \geq 1, a \geq 0 \) an \((r, r+a)\)-factor of a graph \( G \) is a spanning \((r, r+a)\)-subgraph of \( G \). An \((r, r+a)\)-factorization of a graph \( G \) is a decomposition of \( G \) into edge-disjoint \((r, r+a)\)-factors.

We prove a number of results about \((r, r+a)\)-factorizations of \((d, d+s)\)-pseudographs (multigraphs with loops permitted). For example, for \( t \geq 1 \) let \( \pi(r, s, a, t) \) be the least integer such that, if \( d \geq \pi(r, s, a, t) \) then every \((d, d+s)\)-pseudograph \( G \) has an \((r, r+a)\)-factorization into \( x \) \((r, r+a)\)-factors for at least \( t \) different values of \( x \). Then we show that, if \( r \) and \( a \) are even, then

\[
\pi(r, s, a, t) = r \left\lceil \frac{tr + s - 1}{a} \right\rceil + (t - 1)r .
\]

We use this to give bounds for \( \pi(r, s, a, t) \) when \( r \) and \( a \) are not both even. Finally we consider the corresponding functions for multigraphs without loops, and for simple graphs.

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