

Graph Covering by Shortest Paths

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We consider a problem motivated by computer network monitoring and sampling. A network can be modeled as a connected, undirected graph G . Let $I(u,v)$ be the intersection of edge sets on all shortest paths between vertices u and v in G . Let $U(u,v)$ be the union of such edge sets. Let $I(u)$ be the union of edges in $I(u,v)$, over all vertices v in G . Let $U(u)$ be the union of $U(u,v)$ over all vertices v . We define two new shortest path covering problems, $SPCI(G)$ and $SPCU(G)$. The problems accept a graph G as an input and ask for minimum size vertex sets that include all edges of G in the union of $I(u)$ and $U(u)$, respectively, over all vertices u in the sets. We prove that the decision versions of both problems are NP-Complete. We also present efficient solutions for both problems on several restricted classes of graphs.

Keywords: graph theory, graph cover, shortest path, NP complete