Probe Interval Orders, and Unit Probe Interval Graphs

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A graph $G$ is a probe interval graph if there is a vertex partition $V(G) = P \cup N$ and an interval $I_u$ assigned to each vertex $v$ such that $uv \in E(G)$ if and only if $I_u \cap I_v \neq \emptyset$ and one of $u, v$ belongs to $P$. An interval graph is the intersection graph of a collection of intervals of the real line, and enjoy the property that they have a transitive orientation in their complement; whence they are the incomparability graph of an interval order. In this vein, we characterize the probe interval graphs that have a transitively orientable complement (cocomparability probe interval graphs [henceforth cocoPIGs]) and hence give rise to a probe interval order. Our characterization relates cocoPIGs to chordal probe graphs and therefore gives rise to a polynomial recognition algorithm for a non-partitioned graph to be a cocoPIG. We also present a forbidden induced subgraph characterization for bipartite unit probe interval graphs, and report on the characterization of probe interval graphs by forbidden induced subgraphs.

Keywords: Probe interval graph, cocomparability graphs, poset, recognition algorithm.