On Chromatic Connection in Graphs

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Let $G$ be an edge-colored connected graph. A path $P$ is a proper path in $G$ if no two adjacent edges of $P$ are colored the same. If $P$ is a proper $u - v$ path of length $d(u,v)$, then $P$ is a proper $u - v$ geodesic. An edge coloring $c$ is a proper-path coloring of a connected graph $G$ if every pair $u,v$ of distinct vertices of $G$ are connected by a proper $u - v$ path in $G$, while $c$ is a strong proper coloring if every two vertices $u$ and $v$ are connected by a proper $u - v$ geodesic in $G$. The minimum number of colors required for a proper-path coloring and strong proper coloring of $G$ is called the proper connection number and strong connection number of $G$, respectively. These concepts were inspired by the well-studied concepts of rainbow colorings and strong rainbow colorings of a connected graph. We investigate relationships among these four edge colorings as well as the well-known proper edge colorings of graphs and the chromatic index of a graph. Furthermore, new results and open questions are presented in this area of research.

Keywords: proper path and geodesic, chromatic connection.