Combinatorial methods in tropical linear algebra

Alexander Guterman, Lomonosov Moscow State University

Tropical algebra (sometimes called max algebra) is a set of real numbers equipped with the maximum operation instead of usual addition and addition instead of usual multiplication. Under these operations this is an algebraic structure called a semiring. Semirings naturally appear in different problems of combinatorics, communication complexity, scheduling theory, optimization, dynamical systems, etc. Semiring arithmetics allows to reduce non-linear problems to the linear problems but over semirings. To investigate these problems it is necessary to develop linear algebra over semirings and tropical linear algebra is sometimes called the linear algebra of combinatorics. We plan to introduce several tropical rank functions: factor rank, term rank, tropical rank, determinantal rank, Gondran-Minoux rank. Then we discuss combinatorial methods to investigate these rank functions. In particular we discuss recently introduced method of tropical matrix patterns, which allows to reduce several problems on matrix ranks to the problem of existence of even cycles in \( k \)-connected graphs. We describe our recent research results on this topic joint with Marianne Akian, LeRoy Beasley, Stephane Gaubert, and Yaroslav Shitov.

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