GROUPS WITH FEW ISOMORPHISM TYPES OF DERIVED SUBGROUPS

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By a derived subgroup in a group $G$ is meant the commutator subgroup $H'$ of a subgroup $H$ of $G$. We define $C(G)$ to be the set of derived subgroups in the group $G$.

We are interested in the questions how important is the subset $C(G)$ in the lattice $S(G)$ of all subgroups of $G$, and what are the consequences for the structure of $G$ if conditions are imposed on the set $C(G)$. For example groups with $C(G)$ finite have been studied in 1 and 3.

We denote by $D_n$ the class of groups in which the number of the isomorphism types of derived subgroup is at most $n$ and we write $D = \bigcup_{n \in \mathbb{N}} D_n$.

In this talk we give some general results about the classes $D_n$ and $D$. Then we focus our attention on the class $D_2$. We describe in a precise way some large classes of $D_2$-groups. We obtain a complete description of finite $D_2$-groups, the solution leads an interesting number theoretic problem. In addition we obtain detailed information about soluble groups in $D_2$, especially those of finite rank, where algebraic number fields play an important role.

Finally we study finite $D_3$-groups and we give a complete description in the nilpotent case.

REFERENCES